

# MAGNITUDE OF DEVIATORIC TERMS IN VERTICALLY AVERAGED FLOW EQUATIONS

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## ABSTRACT

The depth-integrated momentum and kinetic energy equations contain velocity correlation terms that involve products of local deviations in velocity components about depth-averaged values. Based on velocity data obtained from North Boulder Creek, Colorado, a simple scaling analysis suggests that certain of these terms, which normally can be neglected in the case of smooth channels, can be significant parts of the momentum and energy balances in steep, rough channels owing to the occurrence of non-logarithmic velocity profiles. A linearized version of the kinetic energy equation suggests that, for flow accelerations over small-amplitude bed forms, the energy of the mean motion is spatially partitioned between a form involving the depth-averaged velocity and a form involving the deviatoric part of the velocity profile; this partitioning is associated with spatial variations in the uniformity of the vertical profile of the streamwise velocity. These points are consistent with published flume measurements involving flow over sand-roughened dunes, and with published field measurements of flow over a gravel bar. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: channel roughness; flow acceleration; kinetic energy; momentum equations; North Boulder Creek

## INTRODUCTION

Local, vertical profiles of streamwise velocity in steep mountain streams often are non-logarithmic in form (e.g. Marchand *et al.*, 1984; Jarrett, 1990; Byrd *et al.*, 2000), owing in part to bed roughness produced by coarse gravel and cobbles that extend into the water column a distance on the order of one-tenth of the flow depth or more. Following a companion paper (Byrd *et al.*, 2000), in which we considered the practical task of estimating depth-averaged velocities, we turn here to a related issue that also arises from the presence of non-logarithmic profiles in rough channels. Namely, we consider the significance of velocity correlation terms in the depth-integrated momentum and kinetic energy equations; these terms involve products of local deviations in time-averaged velocity components about depth-averaged values.

It is often advantageous to neglect these velocity correlation terms in the momentum equations for flow in open channels, but this is acceptable only if these terms are small for the conditions of interest. Specifically, these correlation terms can justifiably be neglected for flow in straight channels with small roughness, but they may contribute significantly to the momentum balance in other situations, such as steep streams with large roughness. To illustrate this point, we describe a large set of detailed velocity measurements from North Boulder Creek, Colorado, obtained specifically to evaluate the magnitudes of the velocity correlations  $\overline{u'^2}$  and  $\overline{u'^3}$ , where  $u'$  denotes a local deviation in the streamwise (time-averaged) velocity component about the depth-averaged value  $\bar{u}$ , and the overbar denotes a depth-averaged quantity (see below). For comparison, we also

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examine the laboratory measurements of Nelson *et al.* (1993) involving flow over sand-roughened dunes, and the field measurements of Whiting and Dietrich (1991) involving flow over a gravel bar.

### VELOCITY PROFILES AND CORRELATION TERMS

The motivation for our analysis can be concisely illustrated by considering the depth-integrated momentum equation. The analysis pertains to both its streamwise and transverse components in the context of natural channels with two-dimensional bed topography. Nonetheless, in relation to a linear stability analysis of bed forms in rough channels (Furbish *et al.*, 1998), a scaling analysis of terms in the momentum and kinetic energy equations (Furbish, 1998) indicates that particular attention should be given to the velocity correlations  $\overline{u'^2}$  and  $\overline{u'^3}$  appearing in the streamwise components of these equations. Moreover, flows over one-dimensional bed forms (that is, where the bed topography varies only in the streamwise direction), for example dunes, are also of interest (e.g. Nelson *et al.*, 1993). For these reasons, and for the sake of simplicity, we focus here on the streamwise component of the momentum equation, neglecting transverse velocity and stress terms (although we do not suggest that these quantities are unimportant with two-dimensional bed topographies).

Consider a straight channel whose bed topography varies only in the streamwise direction (e.g. Nelson *et al.*, 1993). The  $x$ -axis is parallel to the streamwise direction, positive downstream, and inclined at an angle  $\theta$  from the horizontal; the  $z$ -axis is nearly vertical and positive upward. Assuming hydrostatic conditions and a stress-free water surface, and letting an overbar denote a depth-averaged quantity, the depth-integrated equation of momentum for steady flow is:

$$\bar{u} \frac{\partial \bar{u}}{\partial x} + \frac{1}{h} \frac{\partial}{\partial x} \left( h \overline{u'^2} \right) + g \frac{\partial \zeta}{\partial x} + g \sin \theta - \frac{1}{\rho h} \left[ \frac{\partial}{\partial x} (h \overline{\tau_{xx}}) + (\tau_{xx})_{\eta} \frac{\partial \eta}{\partial x} - (\tau_{zx})_{\eta} \right] = 0 \quad (1)$$

where  $u$  is the local (time-averaged) streamwise velocity component,  $u'$  is the local deviation in the streamwise velocity component about the depth-averaged value  $u$ ,  $h$  is the local flow depth,  $\zeta$  is the local coordinate of the water surface,  $\eta$  is the local coordinate of the bed surface,  $\tau_{ij}$  is a deviatoric stress component where the subscript  $\eta$  denotes that the stress component is evaluated at the bed,  $\rho$  is the density of water, and  $g$  is acceleration due to gravity.

In using Equation 1 to describe open channel flow, to retain the term in involving  $\overline{u'^2}$  generally requires introducing an additional equation that contains this correlation quantity. For this reason, we also consider the kinetic energy equation of the time-averaged motion, formed as the dot product of the time-averaged velocity vector and the time-averaged momentum equation, where the resulting scalar expression is then depth-integrated. Assuming hydrostatic conditions, and making use of simplifications that arise from the depth-integrated momentum and continuity equations:

$$\frac{2\overline{u'^2}}{\bar{u}} \frac{\partial \bar{u}}{\partial x} + \frac{\partial}{\partial x} \overline{u'^2} + \frac{1}{h\bar{u}} \frac{\partial}{\partial x} \left( h \overline{u'^3} \right) - \frac{2}{\rho \bar{u}} \overline{u' \frac{\partial \tau'_{xx}}{\partial x}} - \frac{2}{\rho \bar{u}} \overline{u' \frac{\partial \tau'_{zx}}{\partial z}} = 0 \quad (2)$$

where  $\tau'_{ij}$  denotes the local deviation in the deviatoric stress component about its depth-averaged value  $\overline{\tau_{ij}}$ . (Also note that  $\partial \tau'_{zx} / \partial z = \partial \tau_{zx} / \partial z$ .) A derivation of the full two-dimensional form of Equation 2 is presented elsewhere (Furbish, 1998).

The term involving  $\overline{u'^2}$  in Equation 1 typically can be neglected (e.g. Smith and McLean, 1984 (with reference to Dietrich, 1982); Struiksma and Crosato, 1989), or is neglected for simplicity (e.g. Furbish, 1993), although it can be incorporated within numerical treatments (e.g. Nelson and Smith, 1989). This is based on the assumption that  $\overline{u'^2} = \overline{u^2} + \overline{u'^2} \approx \overline{u^2}$ . Justification for this appeals in part to the presence of a logarithmic, or approximately logarithmic, velocity profile with small  $z_0/h$ , where  $z_0$  is the nominal height above the bed at which  $u = 0$ , such that the ratio  $\overline{u'^2}/\overline{u^2}$  is on the order of 0.01 or smaller. Assuming a logarithmic profile, for

example, the ratio  $\overline{u'^2}/\overline{u}^2$  is very well approximated by:

$$\frac{\overline{u'^2}}{\overline{u}^2} \approx \frac{\frac{z_0}{h} \ln z_0 \left( \ln h + \ln \frac{h}{z_0} \right) + 1}{(\ln h)^2 - \ln z_0 \left( \ln h + \ln \frac{h}{z_0} \right) - 2 \ln \frac{h}{z_0} + 1} \quad (3)$$

This ratio varies from about 0.015 to 0.061 over two orders of magnitude of  $z_0/h$  (0.0001 to 0.01), for typical values of  $h$ . (The derivation of Equation 3 does not assume that  $z_0 \ll h$ , and thus involves depth integrations between the limits  $z = z_0$  and  $z = h$ .)

Because most of the shear of the time-averaged motion occurs near the bed in the case of a logarithmic profile, most of the contribution to  $\overline{u'^2}$  is associated with the lower 10 per cent or so of the flow column, and in the limit of uniform  $u(z)$ ,  $\overline{u'^2}$  goes to zero. When  $z_0/h$  approaches, say, 0.02 or more, however, the ratio in Equation 3 may no longer be negligibly small, with the implication that the term involving  $\overline{u'^2}$  in Equation 1 can become a significant part of the momentum balance in rough channels (Furbish, 1998).

Local streamwise velocity profiles in rough channels, moreover, typically are non-logarithmic in form (e.g. Marchand *et al.*, 1984; Jarrett, 1990; Byrd, 1997; Byrd *et al.*, 2000). In this vein, Nelson *et al.* (1991) extended the analysis of Wiberg and Smith (1991) to suggest that spatially averaged velocity profiles can approach a linear form with large relative roughness. In this limit, the ratio  $\overline{u'^2}/\overline{u}^2$  is very well approximated by:

$$\frac{\overline{u'^2}}{\overline{u}^2} = \frac{1}{3} \frac{h^3}{(h - z_0)^3} \approx \frac{1}{3} \quad (4)$$

where it is assumed that  $z_0$  can be defined as in the case of a logarithmic profile. In this situation, because the shear of the time-averaged motion occurs uniformly over  $z$ , the contribution to  $\overline{u'^2}$  is associated with much of the profile, mostly over the lower and the upper thirds of the flow column.

Data for a rough channel presented below indicate that the ratio  $\overline{u'^2}/\overline{u}^2$  is on the order of 0.1, but locally can be as large as 0.4 or more. In view of this, to retain the term involving  $\overline{u'^2}$  within the momentum equation, we have in related work appealed to the kinetic energy equation (Equation 2) to obtain closure (Byrd, 1997; Furbish, 1998). This, in turn, introduces additional terms whose magnitudes must be evaluated. A simple scaling analysis presented below suggests that the third, fourth and fifth terms in Equation 2 are small relative to the first two. In this regard, the fourth term is negligible for small values of  $\partial \tau'_{xx}/\partial x$ , that is, for small-amplitude bed forms; and the fifth term vanishes in either of the limiting cases that  $\tau_{zx}$  is a constant or varies linearly with depth. With regard to the third term, and again momentarily assuming a logarithmic profile, the ratio  $|\overline{u'^3}/\overline{u}^3|$  remains less than 0.01 over two orders of magnitude of  $z_0/h$  (0.0001 to 0.01), for typical values of  $h$ . Because  $u'^3$  is an odd function, the positive and negative contributions to  $\overline{u'^3}$  tend to be cancelling. Note that, in the limit of a linear profile, whereas  $\overline{u'^2}$  takes on a maximum value (for given  $\overline{u}$ ),  $\overline{u'^3}$  vanishes. We evaluate the magnitude of the quantity  $\overline{u'^3}$  below; our data indicate that the ratio  $|\overline{u'^3}/\overline{u}^3|$  is on the order of 0.01.

## FIELD AND LABORATORY OBSERVATIONS

Several sets of data are considered here. We use field measurements from North Boulder Creek to estimate magnitudes of the correlation velocities  $u'^2$  and  $\overline{u'^3}$  associated with non-logarithmic velocity profiles over a rough channel bed, independently of the spatial derivatives in Equations 1 and 2. (Our data from North Boulder Creek are not amenable to directly estimating the terms in these equations.) These estimates can then be used in a simple scaling of certain terms in Equations 1 and 2 for the rough bed conditions of North Boulder Creek. We then turn for comparison to the laboratory measurements of Nelson *et al.* (1993). The bed conditions in these experiments, involving sand-roughened one-dimensional dunes, are quite different from the coarse particle roughness in North Boulder Creek. So, although these experiments do not directly pertain to large relative (skin) roughness, they nonetheless nicely illustrate the velocity profile structure associated with flow accelerations over bed forms, and the partitioning of kinetic energy among terms in Equation 2. In

addition, these experiments provide a useful contrast for illustrating differences in the magnitudes of correlation velocities over a rough bed (North Boulder Creek) versus a smooth skin roughness. We also briefly consider the field measurements of Whiting and Dietrich (1991) to qualitatively illustrate certain points about spatial variations in velocity profiles for a rough gravel-bed channel.

### North Boulder Creek

The physical setting of North Boulder Creek is described elsewhere (Furbish, 1987, 1993). The average bankfull width of the study reach is about 2.9 m, and, except for one short bend, the reach is approximately straight. Its average gradient is 0.034. Channel banks are steep, often nearly vertical. The bed consists of coarse gravel and cobbles; the average diameter is about 0.12 m (with variance of 0.0056 m<sup>2</sup>) based on a Wolman count of 204 particles on the bed of a large bend immediately downstream of the study reach. Judging from surveys of transverse bed and water-surface profiles at 120 cross-sections equally spaced at 1 m intervals along the channel, the ratio of roughness height to flow depth typically is on the order of 0.1 (or smaller) at bankfull stage. Nonetheless, large immobile boulders, areally covering 1 or 2 per cent of the total bed area, locally protrude far into (or above) the flow.

During June 1995, vertical profiles of time-averaged streamwise velocity at 60 positions were obtained. (These measurements were obtained specifically to examine the magnitudes of the quantities  $\overline{u'^2}$  and  $\overline{u'^3}$ , and are separate from velocity measurements obtained at the 120 sections, to be described elsewhere.) Each profile was based on measurements at eight to 10 approximately evenly spaced positions in the water column, depending on local depth, which varied from 0.22 to 0.51 m. The details of these measurements are described in Byrd *et al.* (2000).

Only about 10 per cent of the measured profiles could be classified as being logarithmic, and the form of the spatially averaged profile was qualitatively consistent with the theoretical profiles for large relative roughness suggested by Wiberg and Smith (1991) and Nelson *et al.* (1991). Depth-averaged quantities were obtained using Riemann summations with interval limits located at midpoints between adjacent data positions. (This choice of interval limits was a matter of convenience, and coincides with datum-centred intervals over most of the flow depth.) The ratio  $\overline{u'^2}/\bar{u}^2$  varied from 0.02 to 0.46 about an average value of 0.11; the ratio  $\overline{u'^3}/\bar{u}^3$  varied from -0.06 to 0.07 about an average value of -0.01. The distribution of  $\overline{u'^2}/\bar{u}^2$  is skewed, with a variance of 0.0059; the distribution of  $\overline{u'^3}/\bar{u}^3$  is approximately symmetrical, with a variance of 0.0004. The data moreover indicate that  $\overline{u'^2}/\bar{u}^2$  and  $\overline{u'^3}/\bar{u}^3$  are independent of  $\bar{u}$  (Figure 1).

These results can be used directly in a simple scaling of the first two terms in Equation 1 and the five terms in Equation 2. Introducing the following dimensionless quantities denoted by circumflexes:

$$\frac{\partial}{\partial x} = \frac{\Gamma}{H} \frac{\partial}{\partial \hat{x}}; \quad \frac{\partial}{\partial z} = \frac{1}{H} \frac{\partial}{\partial \hat{z}}; \quad h = H\hat{h}; \quad \bar{u} = U\hat{u}; \quad \tau'_{ij} = \rho\Gamma U^2 \hat{\tau} \quad (5)$$

and defining  $\varepsilon = O(0.1)$ , it is assumed that:

$$\overline{u'^2} = \varepsilon U^2 \hat{u}^2; \quad \overline{u'^3} = \varepsilon^2 U^3 \hat{u}^3 \quad (6)$$

where  $U$  and  $H$  denote a characteristic velocity and depth, taken here as the spatial (reach) averages of  $\bar{u}$  and  $h$ , and  $\Gamma$  is a dimensionless coefficient of friction defined in the following specification of the boundary stress:

$$(\tau_{zx})_\eta = \rho\Gamma\bar{u}^2 \quad (7)$$

As a point of reference, the coefficient  $\Gamma$  typically is  $O(0.01) = O(\varepsilon^2)$  for gravel-bed rivers, but it can become as large as  $O(0.1) = O(\varepsilon)$  for steep, rough channels. Estimates of  $\Gamma$  for the North Boulder Creek study reach vary from about 0.08 at bankfull flow with  $H = 0.42$  m to 0.16 with  $H = 0.24$  m (Furbish *et al.*, 1998). The factor  $H/\Gamma$  in Equations 5 represents a relaxation length of the mean flow; it is a measure of the distance over which a perturbation in the flow persists downstream. This distance serves as a convenient streamwise length

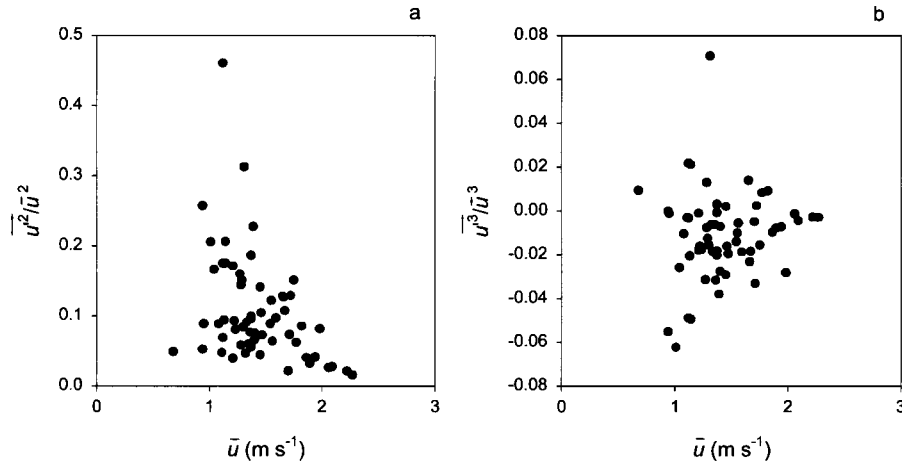


Figure 1. Plots of the ratios  $\overline{u'^2}/\bar{u}^2$  (a) and  $\overline{u'^3}/\bar{u}^3$  (b) versus  $\bar{u}$  illustrating that these ratios are effectively independent of  $\bar{u}$

scale because it intrinsically characterizes the flow yet is independent of bed forms (and their associated length scales).

In their scaling analysis, Nelson and Smith (1989, p. 78) effectively assume that  $|\overline{u'}| = \varepsilon \bar{u}$  which, for a logarithmic profile, is generally valid above a height that is on the order of 0.1 of the flow depth. The scaling of  $\overline{u'^2}$  and  $\overline{u'^3}$  in Equations 6, suggested by our field data, therefore is approximately equivalent to assuming that  $|\overline{u'}| = \sqrt{\varepsilon} \bar{u}$ , which represents a conservative estimate of the magnitude of  $|\overline{u'}|$  in the case of a logarithmic profile. With  $|\overline{u'}| = \varepsilon U \hat{u}$ , and using Equations 5 and 6, the first two terms in Equation 1 and the five terms in Equation 2 may be written as:

$$\hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \varepsilon \frac{1}{\hat{h}} \frac{\partial}{\partial \hat{x}} (\hat{h} \hat{u}^2) + \dots \quad (8)$$

$$\varepsilon 2 \hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \varepsilon \frac{\partial}{\partial \hat{x}} (\hat{u}^2) + \varepsilon^2 \frac{1}{\hat{h} \hat{u}} \frac{\partial}{\partial \hat{x}} (\hat{h} \hat{u}^3) - \sqrt{\varepsilon} \Gamma \frac{2}{\hat{u}} \frac{\partial \hat{\tau}}{\partial \hat{x}} - \sqrt{\varepsilon} \frac{2}{\hat{u}} \frac{\partial \hat{\tau}}{\partial \hat{z}} = 0 \quad (9)$$

where the quantity  $\Gamma U^2/H$  has been divided out of all terms. This scaling suggests that the lowest relative order at which each of the terms in Equations 8 and 9 enters the momentum or energy balance is indicated by its relative order implied by  $\varepsilon$ ,  $\varepsilon^2$  or  $\sqrt{\varepsilon} \Gamma$ , except for the fifth term in Equation 9. This fifth term enters at a relative order no lower than  $\varepsilon \sqrt{\varepsilon}$ , at least for small bed undulations.

Specifically, consider the depth-averaged quantity:

$$\overline{u' \frac{\partial \tau'_{zx}}{\partial z}} = \overline{u' \frac{\partial \tau_{zx}}{\partial z}} \quad (10)$$

appearing in the fifth term of Equation 2. By definition, when  $\partial \tau_{zx}/\partial z = \text{const}$ , as in the case of uniform flow, Equation 10 becomes:

$$\text{const} \cdot \overline{u'} = 0 \quad (11)$$

Therefore, the fifth term in Equation 2 is non-zero only in the case where  $T(z) = \partial \tau_{zx}/\partial z$  varies with  $z$ .

Writing  $T = T + T'$ , Equation 10 may be written as:

$$\overline{u'(T + T')} = \overline{u'T'} \quad (12)$$

and it is reasonable to assume that the magnitude of the depth-averaged product in Equation 12 is of the same order as the product  $|\overline{u'}| |\overline{T'}|$ . If we further assume that, at least for small bed undulations,  $|\overline{T'}| = \varepsilon \rho \Gamma U^2 \hat{\tau} / H$ , then the fifth term in Equation 9 (and in Equation 14; see below) is multiplied by an additional  $\varepsilon$ . In addition, for flows whose response to an undulating bed topography involves an inviscid-like behaviour over the upper portion of the water column, as in the experiments of Nelson *et al.* (1993), and probably manifest in the velocity measurements of Whiting and Dietrich (1991) (see next section), then this portion of the column, where the stress gradient is approximately zero, contributes negligibly to the fifth term in Equation 2.

The appearance of  $\varepsilon$  in Equation 8 suggests that the quantity  $u'^2$  may be sufficiently large in some situations to justify including the second term in the momentum balance (Equation 1), such as flows in rough channels. In this case, the quantity  $u'^3$  is likely to be sufficiently small that the third term in the kinetic energy balance (Equation 2) can be neglected. For a later comparison regarding this point, we also list here analogous expressions for Equations 8 and 9 consistent with conditions in smoother channels in which the velocity profile is approximately logarithmic, where it may be assumed that  $u'^2 = \varepsilon^2 U^2 \hat{u}^2$ :

$$\hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \varepsilon^2 \frac{1}{\hat{h}} \frac{\partial}{\partial \hat{x}} (\hat{h} \hat{u}^2) + \dots \quad (13)$$

$$\varepsilon^2 2\hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \varepsilon^2 \frac{\partial}{\partial \hat{x}} (\hat{u}^2) + \varepsilon^2 \frac{1}{\hat{h}\hat{u}} \frac{\partial}{\partial \hat{x}} (\hat{h}\hat{u}^3) - \varepsilon \Gamma \frac{2}{\hat{u}} \frac{\partial \hat{\tau}}{\partial \hat{x}} - \varepsilon \frac{2}{\hat{u}} \frac{\partial \hat{\tau}}{\partial \hat{z}} = 0 \quad (14)$$

The appearance of  $\varepsilon^2$  in Equation 13 indicates that the term in Equation 1 involving  $\overline{u'^2}$  contributes negligibly to the momentum balance, wherein a consideration of Equation 2 becomes unnecessary. In addition, the fifth term in Equation 14 enters at a relative order no lower than  $\varepsilon^2$  for reasons given following Equation 12.

#### *Related flume measurements and field observations*

The effect of neglecting the term in Equation 1 involving  $\overline{u'^2}$  generally is to underestimate the magnitude of convective accelerations. In this regard, a linearized version of Equation 2 provides an interesting, approximate interpretation of the vertical structure of these accelerations. Assume that  $\bar{u}$  and  $\overline{u'^2}$  can be expressed as:

$$\bar{u} = U + u^*; \quad \overline{u'^2} \equiv \nu = \Upsilon + \nu^* \quad (15)$$

where  $u^*$  and  $\nu^*$  denote small perturbations in  $\bar{u}$  and  $\nu \equiv \overline{u'^2}$  about the reach-averaged values  $U$  and  $\Upsilon$ . Using these definitions, and momentarily neglecting the third, fourth and fifth terms in Equation 2, this equation is linearized to:

$$\frac{2\Upsilon}{U} \frac{\partial u^*}{\partial x} + \frac{\partial \nu^*}{\partial x} = 0 \quad (16)$$

This implies that:

$$\frac{2\Upsilon}{U} u^* + \nu^* = \text{const} \quad (17)$$

which describes a partitioning of kinetic energy into a form involving the depth-averaged velocity

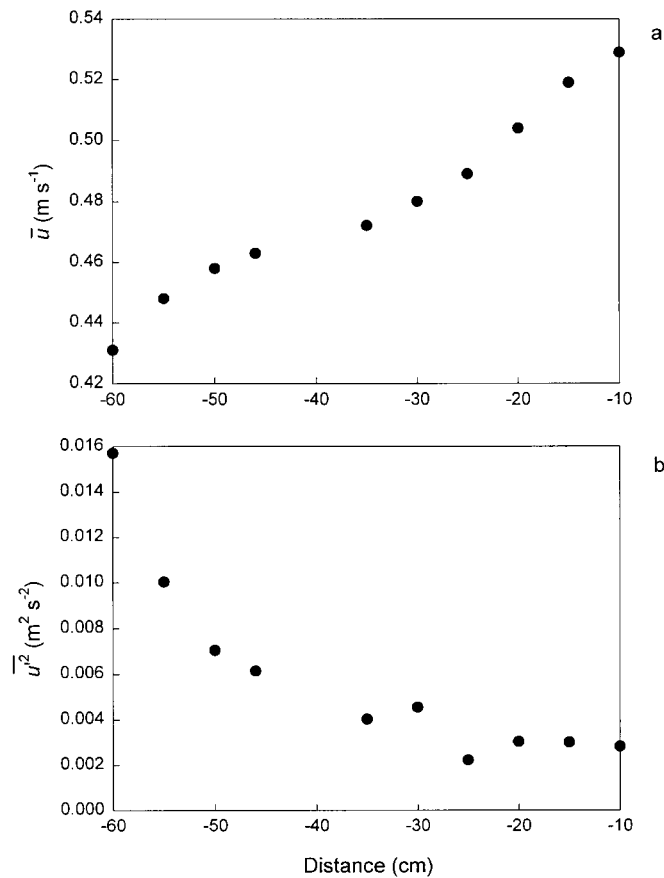


Figure 2. Plot of depth-averaged velocity  $\bar{u}$  (a) and correlation velocity  $\overline{u'^2}$  (b) versus streamwise distance for flume experiments of Nelson *et al.* (1993)

perturbation  $u^*$ , and a form involving the deviatoric part of the velocity profile,  $u'$ , contained in the perturbation  $v^*$ . A positive value of  $\partial u^*/\partial x$  in Equation 16, for example, is compensated by a negative value of  $\partial v^*/\partial x$ , such that  $v$  decreases downstream. This generally means that the local velocity  $u(z)$  must become more uniform over  $z$  in the direction of  $x$ , such that  $\overline{u'^2}$  also decreases in the direction of  $x$ . Due to the no-slip condition, shear of the time-averaged motion becomes increasingly concentrated near the bed, inasmuch as  $u(z)$  increases monotonically with  $z$ .

These points can be loosely compared with the experimental results of Nelson *et al.* (1993). These experiments involve sand (skin) roughness on asymmetrical dunes, so the relative magnitudes of the terms in Equation 2 are likely to be different, at least locally, from those associated with flow over rougher beds without dunes. Nonetheless, because the energy transfers described by Equation 2 are generally valid when the hydrostatic approximation is satisfied, these experiments nicely illustrate certain consistencies with the points above. In particular, refer to figure 2 (run 1) of Nelson *et al.* (1993), which illustrates streamwise acceleration of flow onto the stoss of a two-dimensional dune. The height and length of the dune are such that the inviscid response of the flow to the dune topography extends throughout the water column, manifest as an overall acceleration of the flow. Shear becomes increasingly concentrated near the bed, manifest as a streamwise strengthening of the gradient  $\partial u/\partial z$  within the lower part of the wake region downstream of the dune crest, noticeably between streamwise positions  $-55$  cm and  $-40$  cm, and also within the internal boundary layer very close to the bed, particularly between streamwise positions  $-50$  cm and  $-10$  cm. Also note that, at about one dune height above the dune crests, the Reynolds stress  $\tau_{zx}$  is either approximately

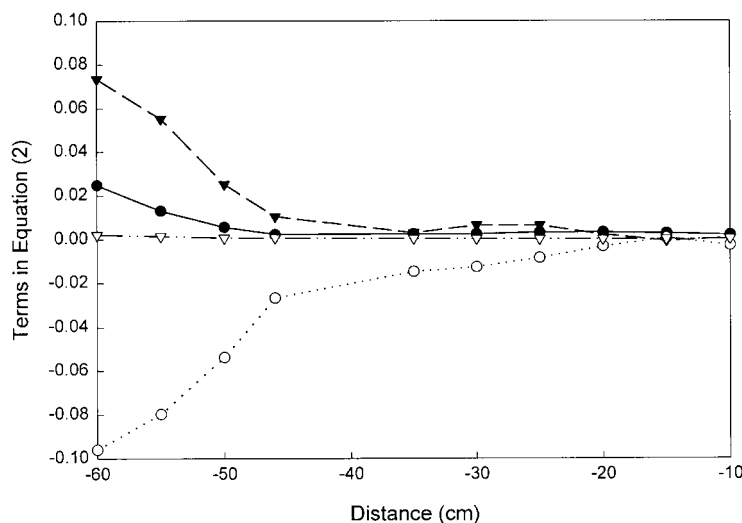


Figure 3. Plot of estimated values of terms in Equation 2 versus streamwise distance for flume experiments of Nelson *et al.* (1993): ●, first term; ○, second term; ▼, third term; ▽, fifth term

uniform and small, or varies linearly with  $z$  (refer to figure 3 b and figure 3 d of Nelson *et al.*, 1993). At least within the upper part of the flow, the gradient  $\partial \tau_{zx} / \partial z$  therefore contributes little to the fifth term in Equation 2.

We digitized values of  $u$  and  $\tau_{zx}$  from smooth curves fitted to the data for ten of the measurement profiles of Nelson *et al.* (1993), then calculated values of  $\bar{u}$ ,  $u'$ ,  $\overline{u'^2}$ ,  $\overline{u'^3}$  and  $\overline{u' \partial \tau_{zx}' / \partial z}$ . As a check on our computations of  $\bar{u}$ , the depth-integrated continuity equation,  $\partial (h\bar{u}) / \partial x = 0$ , requires that the product  $h\bar{u} = \text{const}$ ; computed values of  $h\bar{u}$  varied less than 3 per cent about a constant value.

Consistent with the approximate interpretation of Equation 16, as  $\bar{u}$  increases with  $x$  over the dune stoss,  $\overline{u'^2}$  systematically decreases (Figure 2). The increase in  $\bar{u}$  is mostly due to the inviscid response of the flow, particularly in the upper part of it. In addition, vertical velocities near the bed strengthen over the stoss (see figure 2c of Nelson *et al.*, 1993). By continuity, streamwise velocities near the bed increase in absence of a substantial water-surface response; then, a streamwise concentration of shear occurs near the bed, and  $\overline{u'^2}$  decreases with increasing uniformity of the profile as streamwise velocities near the bed become similar in magnitude to those in the upper inviscid region.

We used simple finite-differencing operators to roughly estimate the local streamwise gradients  $\partial \bar{u} / \partial x$  and  $\partial \overline{u'^2} / \partial x$  in Figure 2, from which the magnitudes of terms in Equation 2 could be obtained (Figure 3; curves in this figure are based on a simple three-point moving-average filter of derivative quantities). The first term in Equation 2 decreases smoothly between  $-60$  cm and  $-45$  cm to an approximately constant value of  $0.005$  (Figure 3). The second term increases smoothly over this distance to a value of about  $-0.01$ , but continues to approach zero over the downstream  $10$  cm. The third term decreases to a value of about  $0.007$ , then approaches zero over the downstream  $10$  cm. The fourth term was not estimated, as data involving  $\tau_{xx}$  for run 1 are not provided in Nelson *et al.* (1993). The fifth term is positive and small over the measurement distance.

The estimated values of terms in Equation 2 (Figure 3) approximately sum to zero at  $-60$  cm, and from  $-30$  cm to  $-10$  cm. A significant deficit occurs, however, in the vicinity of  $-50$  cm, where this sum is as much as 36 per cent of the largest (second) term. This is probably attributable, in part, to errors in our digitizing the data and, possibly, to a significant contribution by the fourth term in Equation 2. Judging from profiles of  $\tau_{xx}$  obtained for run 4 (figure 4a of Nelson *et al.*, 1993), the largest values of  $\partial \tau_{xx} / \partial x$  occur in this region (around  $-50$  cm) near the bed. This deficit also may be attributable, in part, to the fact that Equation 2 provides an inadequate description of the kinetic energy balance immediately downstream of the dune lee due to the effects of flow separation and strong vertical motions, where the hydrostatic approximation, assumed in obtaining Equation 2, is unlikely to be valid. In this regard, the large relative roughness represented by the



dune height-to-depth ratio (approximately 0.19) must be regarded as representing a severe test of the depth-integrated balance (Equation 2). Also note that whereas  $\bar{u}^3/\bar{u}^3$  is at least one, and normally two, orders of magnitude smaller than  $\bar{u}^2/\bar{u}^2$ , the overall magnitude of the third term is comparable to that of the first two, consistent with the scaling in Equation 14.

It is also noteworthy that several of the points above are qualitatively consistent with the field measurements of Whiting and Dietrich (1991), which pertain to shoaling of flow onto a large gravel bar. In particular, they note that shoaling leads to a streamwise change in the velocity structure: the position of the maximum velocity within the flow column is displaced downward with shoaling, and is nearer the surface with deepening (refer to figure 8 of Whiting and Dietrich, 1991, p. 790). These measurements pertain to conditions where transverse flows occur, unlike the experiments of Nelson *et al.* (1993) described above. Nonetheless, this effect of shoaling is particularly visible near the channel centreline at cross-sections 7, 8 and 9 where the bar crest has flattened transversely (refer to figure 7 of Whiting and Dietrich, 1991, p. 789); and they point out that 'the highest velocities are below the water surface when depth is decreasing, presumably because rapid shoaling forces substantial near-bed accelerations that cannot be accounted for by lateral flow' (p. 788). Thus, as in the experiments of Nelson *et al.* (1993), and consistent with the approximate interpretation provided by Equation 16, an overall streamwise acceleration associated with shoaling restructures the velocity profile such that shear of the time-averaged motion becomes increasingly concentrated near the bed.

## DISCUSSION AND CONCLUSIONS

Terms in the momentum equations involving correlation quantities such as  $\overline{u'^2}$  can justifiably be neglected in the momentum balance for flows in straight channels with small relative roughness. Our data for North Boulder Creek, however, suggest that the quantity  $\overline{u'^2}$  may be sufficiently large in some situations to justify including the second term involving  $\overline{u'^2}$  in the momentum balance (Equation 1) for flows in rough channels. The significance of this term is that it characterizes streamwise transport of momentum associated with the deviatoric part of the velocity profile  $u(z)$ , in addition to that associated with the depth-averaged velocity  $\bar{u}$ . Retaining this term, however, requires an additional expression involving  $\overline{u'^2}$ , leading to our examination of the kinetic energy equation (Equation 2).

In using Equation 2, however, some ambiguity exists. Insofar as the third, fourth and fifth terms in Equation 2 are much smaller than the first two, then Equation 2 provides adequate closure. That is, when Equation 2 is substituted into Equation 1, the third, fourth and fifth terms in Equation 2, based on Equation 9, contribute to the momentum balance at  $O(\epsilon^2)$  and therefore can be neglected. Based on Equation 14 and our analysis of the experiments of Nelson *et al.* (1993), however, the first three terms in Equation 2 are of the same magnitude for conditions where  $\bar{u}^2/\bar{u}^2 = O(\epsilon^2)$ . If the disparity between the magnitudes of the terms in Equation 2 increases, such that  $\bar{u}^2/\bar{u}^2 = O(\epsilon)$  or larger, the quantity  $\bar{u}^3$  is then likely to be sufficiently small that the third term in the kinetic energy balance (Equation 2) can be neglected. The likelihood of this occurring increases as the velocity profile becomes less uniform (and more linear in form) such that, for given  $\bar{u}$ ,  $\bar{u}^3$  decreases as  $\bar{u}^2$  increases. Our data suggest that this indeed occurs on average with the large relative roughness of North Boulder Creek. The fourth term in Equation 2 is probably locally significant in the vicinity of large isolated bumps, including boulders and short bed forms with large amplitude-to-wavelength values, but otherwise is likely to be negligible. Indeed, the analogous stress term involving  $\tau_{xx}$  in Equation 1 typically can be neglected for this reason. The fifth term in Equation 2 is negligibly small insofar as the local, vertical distribution of stress does not deviate significantly from a linear form. Although we cannot directly evaluate the magnitude of this term from our field data, we can offer the following points.

If the first two terms in Equation 2 are of the same order and balanced, and if the third and fourth terms are small, as suggested by the scaling in Equation 9 together with our field data, then on purely empirical grounds the fifth term on average must also be small. Indeed, this term vanishes with a linear stress profile over a planar bed with uniform roughness (as do all terms in Equation 2); and at least for small-amplitude bed undulations, this and the fourth term probably remain small. Moreover, inasmuch as the response of the flow to an undulating bed topography involves an inviscid-like behaviour over the upper portion of the water

column, as in the experiments of Nelson *et al.* (1993), then this portion of the column, where the stress gradient is approximately zero, contributes negligibly to the fifth term in Equation 2; and any contribution to the fifth term is limited to the lower part of the column where the gradient of the stress profile changes.

Our analysis is also qualitatively consistent with the experimental results of Nelson *et al.* (1993) involving sand (skin) roughness on asymmetrical dunes. In particular, a systematic partitioning of kinetic energy between the first two terms in Equation 2 occurs as flow accelerates over the dune stoss (Figure 2 and Figure 3), such that  $\bar{u}^2$  decreases with increasing uniformity of the profile  $u(z)$  as streamwise velocities near the bed become similar in magnitude to those in the upper inviscid region. The third term in Equation 2 is of the same order as the first two (Figure 3), consistent with the scaling in Equation 14. This occurs not so much because  $\bar{u}^3$  in the third term is large, but rather, because the velocity profiles  $u(z)$  are sufficiently uniform over  $z$  that the quantity  $\bar{u}^2$  in the first two terms is small. We also note that, due to the strong asymmetry and large height-to-depth ratio of the dunes, the velocity and turbulence fields in the dune lees are complex; here, the hydrostatic approximation is not satisfied, and Equation 2 probably provides an inadequate description of the kinetic energy balance.

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